

QCD and Relativistic $O(\alpha_s v^2)$ Corrections to Hadronic Decays of Spin-Singlet Heavy Quarkonia h_c, h_b and η_b

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Abstract

We calculate the annihilation decay widths of spin-singlet heavy quarkonia h_c, h_b and η_b into light hadrons with both QCD and relativistic corrections at order $O(\alpha_s v^2)$ in nonrelativistic QCD. With appropriate estimates for the long-distance matrix elements by using the potential model and operator evolution method, we find that our predictions of these decay widths are consistent with recent experimental measurements. We also find that the $O(\alpha_s v^2)$ corrections are small for $b\bar{b}$ states but substantial for $c\bar{c}$ states. In particular, the negative contribution of $O(\alpha_s v^2)$ correction to the h_c decay can lower the decay width, as compared with previous predictions without the $O(\alpha_s v^2)$ correction, and thus result in better agreement with the measurement.

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I. INTRODUCTION

The inclusive annihilation decay of heavy quarkonium is one of the important issues in heavy quarkonium physics. Theoretically, it is widely accepted that the heavy quarkonium inclusive annihilation decay can be described by nonrelativistic QCD (NRQCD) factorization [1]. In NRQCD, the long-distance effects that cannot be calculated perturbatively are described by the long-distance matrix elements (LDMEs), which are classified in the order of v , the relative velocity of heavy quarks in quarkonium. Experimentally, a large amount of heavy quarkonia data have given more precise measurements for their decay widths and branching ratios (for a review and recent results see, e.g.[2–4]). Therefore, it is necessary to provide more precise theoretical predictions to compare with experimental measurements.

For charmonium, the inclusive annihilation hadronic decay (into gluons and light quark pairs) widths for S , P , D -wave $c\bar{c}$ states are all calculated up to $O(\alpha_s)$ in NRQCD[5–11]. Particularly, for the S -wave state η_c , the $O(\alpha_s v^2)$ corrections have recently been carried out [12], where the short-distance coefficients of $O(v^2)$ are calculated perturbatively to next-to-leading order (NLO) in α_s . After taking the $O(\alpha_s v^2)$ corrections into account, the measurements of η_c decay can be described much better in NRQCD[12]. For the P -wave state h_c , the earlier theoretical result at $O(\alpha_s)$ predicts the hadronic decay width of h_c to be about 0.72 MeV[9], which is a factor of 2 larger than the latest measurements by BESIII, where the central value of the total width is about 0.73 MeV and the hadronic decay branching ratio is about 50%[4]. Thus it is important to do higher order corrections in v to examine whether the gap between theoretical predictions and experimental measurements can be reduced, and this is also an interesting test of the validity of NRQCD factorization.

For bottomonium, the $b\bar{b}$ system, the value of v^2 is about 0.1, which is much smaller than $v^2 \approx 0.3$ for charmonium. It is then expected that the v^2 expansion should be better for bottomonium, thus the v^2 correction for bottomonium is more solid to check NRQCD factorization. Recently, the process $h_b(1P) \rightarrow \eta_b(1S)\gamma$ is measured by the Belle Collaboration. They find the η_b decay width to be about 12.4 MeV and the decay branching fraction of $\mathcal{B}[h_b(1P) \rightarrow \eta_b(1S)\gamma] = 49.2 \pm 5.7_{-3.3}^{+5.6}\%$ [3]. It is tempting to try to explain these data in NRQCD.

In this paper, we will perform the $O(\alpha_s v^2)$ calculations for the spin-singlet P -wave charmonium h_c and bottomonium h_b , and also for the spin-singlet S -wave bottomonium η_b . We

find these corrections are important to understand the measured data. The rest of this paper is organized as follows. In Sec. II we briefly introduce the NRQCD factorization formalism in heavy quarkonium annihilation decays. Then we describe some technical method in calculating $O(\alpha_s v^2)$ short-distance coefficients in Sec. III. The results for S -wave and P wave-states including real and virtual contributions are presented in Sec. IV B. With these results and appropriate estimates of the LDMEs, we discuss the related phenomenology in Sec. V. Finally, we give a brief summary in Sec. VI.

II. NRQCD FACTORIZATION FOR QUARKONIUM DECAY

In this section, we introduce the NRQCD factorization formula for the rates of spin-singlet heavy quarkonium ($\eta_{c,b}$ and $h_{c,b}$) decays to light hadrons. The inclusive annihilation decay width of heavy quarkonium can be factorized by the following formula [1]

$$\Gamma(H) = \sum_n \frac{2 \operatorname{Im} f_n(\mu_\Lambda)}{m_Q^{d_n-4}} \langle H | \mathcal{O}_n(\mu_\Lambda) | H \rangle, \quad (1)$$

where $\operatorname{Im} f_n(\mu_\Lambda)$ is the short-distance (SD) coefficient which can be perturbatively calculated with full QCD Lagrangian. The long-distance matrix elements (LDMEs) $\langle H | \mathcal{O}_n(\mu_\Lambda) | H \rangle$ involve non-perturbative effects and are classified by the relative velocity v of Q and \bar{Q} .

The NRQCD Lagrangian can be derived by integrating out the degrees of freedom of order m_Q , the mass of the heavy quark

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L}. \quad (2)$$

The heavy part of the Lagrangian describes the motions of (anti-)heavy quark in spacetime and is given by

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger (iD_t + \frac{\mathbf{D}^2}{2m_Q}) \psi + \chi^\dagger (iD_t + \frac{\mathbf{D}^2}{2m_Q}) \chi \quad (3)$$

where $\psi(\chi)$ denotes the Pauli spinor field that annihilates (creates) a heavy (anti-)quark, and $D_t(\mathbf{D})$ is the time(space) component of the gauge-covariant derivative D^μ . The light piece of the Lagrangian reads

$$\mathcal{L}_{\text{light}} = -\frac{1}{2} \operatorname{Tr} G^{\mu\nu} G_{\mu\nu} + \sum_{n_f} \bar{q} i \not{D} q \quad (4)$$

where $G^{\mu\nu}$ is the gluon field strength tensor, q is the Dirac spinor field of light quarks and n_f is the number of light flavors. The bilinear Lagrangian term which contains the order v^2 corrections can also be reproduced

$$\begin{aligned}\delta\mathbf{L}_{\text{bilinear}} = & \frac{c_1}{8m_Q^3}\psi^\dagger(\mathbf{D}^2)^2\psi + \frac{c_2}{8m_Q^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi \\ & + \frac{c_3}{8m_Q^2}\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\sigma\psi + \frac{c_4}{2m_Q}\psi^\dagger(g\mathbf{B}\cdot\sigma)\psi \\ & + \text{charge conjugate terms},\end{aligned}\tag{5}$$

where $E^i = G^{0i}$ and $B^i = \frac{1}{2}\epsilon^{ijk}G^{jk}$ are the electric and magnetic components of the gluon field strength tensor $G^{\mu\nu}$, and $c_i = 1 + O(\alpha_s)$, $i = 1, 2, 3, 4$ are the dimensionless coefficients corresponding to each operator.

In order to describe the annihilation decay of quarkonium, a set of local 4-fermion operators \mathcal{O}_i which appear in Eq. (1) are generated. For example, the operator $\psi^\dagger\chi\chi^\dagger\psi$ can annihilate a $Q\bar{Q}$ pair in the $^1S_0^{[1]}$ configuration. In our case, for the $O(\alpha_s v^2)$ calculation of spin-singlet quarkonium decay, the power counting rules [13–15] give the following six operators and LDMEs in Eq. (1):

for S wave,

$$\mathcal{O}(^1S_0^{[1]}) = \psi^\dagger\chi\chi^\dagger\psi,\tag{6a}$$

$$\mathcal{P}(^1S_0^{[1]}) = \frac{1}{2}\psi^\dagger\chi\chi^\dagger(-\frac{i\vec{\mathbf{D}}}{2})^2\psi + \text{h.c.},\tag{6b}$$

for P wave,

$$\mathcal{O}(^1S_0^{[8]}) = \psi^\dagger T^a\chi\chi^\dagger T^a\psi,\tag{7a}$$

$$\mathcal{P}(^1S_0^{[8]}) = \frac{1}{2}\psi^\dagger T^a\chi\chi^\dagger T^a(-\frac{i\vec{\mathbf{D}}}{2})^2\psi + \text{h.c.},\tag{7b}$$

$$\mathcal{O}(^1P_1^{[1]}) = \psi^\dagger(-\frac{i\vec{\mathbf{D}}}{2})\chi\cdot\chi^\dagger(-\frac{i\vec{\mathbf{D}}}{2})\psi,\tag{7c}$$

$$\mathcal{P}(^1P_1^{[1]}) = \frac{1}{2}\psi^\dagger(-\frac{i\vec{\mathbf{D}}}{2})\chi\cdot\chi^\dagger(-\frac{i\vec{\mathbf{D}}}{2})^3\psi + \text{h.c.},\tag{7d}$$

and

$$\langle\mathcal{O}(^{2S+1}L_J^{[1,8]})\rangle_H \equiv \langle H|\mathcal{O}(^{2S+1}L_J^{[1,8]})|H\rangle,\tag{8a}$$

$$\langle\mathcal{P}(^{2S+1}L_J^{[1,8]})\rangle_H \equiv \langle H|\mathcal{P}(^{2S+1}L_J^{[1,8]})|H\rangle.\tag{8b}$$

By counting the dimension of the above operators [1], we can explicitly rewrite factorization Eq. (1) for 1S_0 and 1P_1 states,

$$\Gamma(H(^1S_0) \rightarrow \text{LH}) = \frac{F(^1S_0^{[1]})}{m_Q^2} \langle \mathcal{O}(^1S_0^{[1]}) \rangle_{^1S_0} + \frac{G(^1S_0^{[1]})}{m_Q^4} \langle \mathcal{P}(^1S_0^{[1]}) \rangle_{^1S_0}, \quad (9a)$$

$$\begin{aligned} \Gamma(H(^1P_1) \rightarrow \text{LH}) &= \frac{F(^1S_0^{[8]})}{m_Q^2} \langle \mathcal{O}(^1S_0^{[8]}) \rangle_{^1P_1} + \frac{G(^1S_0^{[8]})}{m_Q^4} \langle \mathcal{P}(^1S_0^{[8]}) \rangle_{^1P_1} \\ &+ \frac{F(^1P_1^{[1]})}{m_Q^4} \langle \mathcal{O}(^1P_1^{[1]}) \rangle_{^1P_1} + \frac{G(^1P_1^{[1]})}{m_Q^6} \langle \mathcal{P}(^1P_1^{[1]}) \rangle_{^1P_1}. \end{aligned} \quad (9b)$$

Through the above factorization formula, the work is simplified by matching full QCD with NRQCD to get the imaginary part of the short-distance (SD) coefficients F and G perturbatively. The skeleton of the matching procedure is given by

$$\text{Im}\mathcal{M}(Q\bar{Q} \rightarrow Q\bar{Q}) \Big|_{\text{pert QCD}} = \sum_n \frac{2 \text{Im}f_n(\mu_\Lambda)}{m_Q^{d_n-4}} \langle Q\bar{Q} | \mathcal{O}_n(\mu_\Lambda) | Q\bar{Q} \rangle \Big|_{\text{NRQCD}}, \quad (10)$$

and the procedure will be discussed in detail in next section.

III. DETAILS IN FULL QCD CALCULATION

A. Kinematics

We investigate the 4-momentum of heavy quarkonium in its rest frame. It is customary to decompose the momenta of Q and \bar{Q} in the following form,

$$p_Q = \frac{1}{2}P + q, \quad (11a)$$

$$p_{\bar{Q}} = \frac{1}{2}P - q, \quad (11b)$$

where

$$P = (2E_{\mathbf{q}}, \mathbf{0}), \quad (12a)$$

$$q = (0, \mathbf{q}), \quad (12b)$$

where $E_{\mathbf{q}} = \sqrt{m_Q^2 + \mathbf{q}^2}$ and satisfies the relation $P \cdot q = 0$.

The treatment of final state phase space of $O(\alpha_s v^2)$ corrections is slightly different from ordinary calculations (i.e. leading order of v calculation). We make the the following rescaling transformation for all external momenta in amplitudes [12, 16],

$$P \rightarrow P' \frac{E_{\mathbf{q}}}{m_Q}, \quad (13a)$$

$$k_f \rightarrow k'_f \frac{E_{\mathbf{q}}}{m_Q}, \quad (13b)$$

but maintain the relative momentum q and loop integral momentum l unchanged. Once we take such a trick, the \mathbf{q}^2 dependence both in phase space and current factor (i.e. $1/(2M)$ where M is the quarkonium mass) can be absorbed into amplitudes, then we can safely take $\mathbf{q} \rightarrow 0$ in these terms and only expand \mathbf{q} in amplitudes. Here we should note that this trick can only work in the case that all the final state partons are massless (i.e. gluons and light quarks), because for the massive partons the on-shell relation under rescaling do not hold.

B. Covariant Projection Method in D-Dimension

We use an equivalent but more efficient method, i.e. the covariant projection method, to calculate the imaginary part of SD coefficients in Eq. (9a) and (9b). In order to get spin-singlet color-singlet and color-octet $Q\bar{Q}$ decay amplitudes, we take the following spin and color projectors onto $Q\bar{Q}$ quark lines [17]:

$$\Pi_0 = \frac{1}{2\sqrt{2}(E_{\mathbf{q}} + m_Q)} \left(\frac{\not{P}}{2} + \not{q} + m_Q \right) \frac{(\not{P} + 2E_{\mathbf{q}})\gamma_5(-\not{P} + 2E_{\mathbf{q}})}{8E_{\mathbf{q}}^2} \left(\frac{\not{P}}{2} - \not{q} - m_Q \right), \quad (14)$$

and

$$\mathcal{C}_1 = \frac{1}{\sqrt{N_c}}, \quad (15a)$$

$$\mathcal{C}_8 = \sqrt{2}\mathbf{T}^c. \quad (15b)$$

Since the projected amplitudes also depend on q , we should make Taylor expansion in powers of q to the required order,

$$\begin{aligned} \mathcal{M}(q) = & \mathcal{M}(0) + \frac{\partial \mathcal{M}(q)}{\partial q^\alpha} \Big|_{q=0} q^\alpha + \frac{1}{2!} \frac{\partial^2 \mathcal{M}(q)}{\partial q^\alpha \partial q^\beta} \Big|_{q=0} q^\alpha q^\beta \\ & + \frac{1}{3!} \frac{\partial^3 \mathcal{M}(q)}{\partial q^\alpha \partial q^\beta \partial q^\gamma} \Big|_{q=0} q^\alpha q^\beta q^\gamma + \dots, \end{aligned} \quad (16)$$

and then make the replacement:

$$q_\alpha q_\beta \rightarrow \frac{\mathbf{q}^2}{D-1} \Pi_{\alpha\beta}, \quad (17a)$$

$$q_\alpha q_\beta q_\gamma q'_\lambda \rightarrow \frac{\mathbf{q}^2 \mathbf{q} \cdot \mathbf{q}'}{D+1} (\Pi_{\alpha\beta} \Pi_{\gamma\lambda} + \Pi_{\alpha\gamma} \Pi_{\beta\lambda} + \Pi_{\alpha\lambda} \Pi_{\gamma\beta}), \quad (17b)$$

where

$$\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{P'_\alpha P'_\beta}{4m_Q^2}, \quad (18)$$

and \mathbf{q}' is the relative momentum corresponding to the complex conjugate of the amplitudes $\mathcal{M}^\dagger(q')$ and P' is the rescaling heavy quarkonium momentum. For example, the third derivative term of \mathcal{M} times the first derivative term of \mathcal{M}^\dagger gives the squared amplitudes term

$$\begin{aligned} & \frac{1}{3!} \frac{\partial^3 \mathcal{M}(q)}{\partial q^\alpha \partial q^\beta \partial q^\gamma} \Big|_{q=0} \frac{\partial \mathcal{M}^\dagger(q')}{\partial q'^\lambda} \Big|_{q'=0} q^\alpha q^\beta q^\gamma q'^\lambda \\ \rightarrow & \frac{1}{3!} \frac{\mathbf{q}^2 \mathbf{q} \cdot \mathbf{q}'}{D+1} (\Pi_{\alpha\beta} \Pi_{\gamma\lambda} + \Pi_{\alpha\gamma} \Pi_{\beta\lambda} + \Pi_{\alpha\lambda} \Pi_{\gamma\beta}) \frac{\partial^3 \mathcal{M}(q)}{\partial q^\alpha \partial q^\beta \partial q^\gamma} \Big|_{q=0} \frac{\partial \mathcal{M}^\dagger(q')}{\partial q'^\lambda} \Big|_{q'=0} \end{aligned} \quad (19)$$

and this term contributes to the SD coefficient of $G(^1P_1^{[1]})$ in Eq. (9b).

IV. PERTURBATIVE QCD RESULTS OF SHORT-DISTANCE COEFFICIENTS

Our techniques of calculating the SD coefficients at $O(\alpha_s v^2)$ are as follows. First we generate Feynman diagrams and amplitudes by **FeynArts** [18, 19], and then calculate the squared amplitudes by self-written **Mathematica** codes, and the phase space integrals are calculated analytically by the method presented in Ref. [8]. Ultra-violet(UV) and infra-red(IR) divergences are regularized by dimensional regularization. The renormalization of UV divergences is taken by re-defining the renormalized heavy quark mass m_Q , heavy quark field ψ_Q , light quark field ψ_q and gluon field A_μ in the on-mass-shell scheme(OS), and the

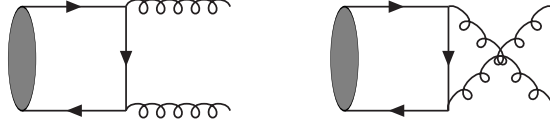


FIG. 1: Born level Feynman diagrams for $^1S_0^{[1]} \rightarrow gg$.

QCD coupling constant g_s in the \overline{MS} scheme,

$$\delta Z_{m_Q}^{OS} = -3C_F \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{UV}} + \frac{4}{3} \right], \quad (20a)$$

$$\delta Z_2^{OS} = -C_F \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4 \right], \quad (20b)$$

$$\delta Z_{2l}^{OS} = -C_F \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right], \quad (20c)$$

$$\delta Z_3^{OS} = \frac{\alpha_s}{4\pi} N_\epsilon \left[(\beta_0 - 2C_A) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \right], \quad (20d)$$

$$\delta Z_g^{\overline{MS}} = -\frac{\beta_0}{2} \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{UV}} + \ln \frac{m_Q^2}{\mu_r^2} \right], \quad (20e)$$

where $N_\epsilon(m_Q) = (\frac{4\pi\mu_r^2}{m_Q^2})^\epsilon \Gamma(1+\epsilon)$ is an overall factor, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient in the β function of QCD, n_f is the active quark flavors, and μ_r is the renormalization scale.

A. Short-Distance Coefficients of S-Wave Quarkonium in Hadronic Decay

Our leading order calculations in α_s give the Born level decay width and its relativistic correction respectively as

$$\Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg) = \frac{4}{3} (4\pi\alpha_s)^2 \frac{\mu_r^{4\epsilon}}{m_Q^2} \Phi_{(2)} (1-\epsilon)(1-2\epsilon) \frac{\langle \mathcal{O}(^1S_0^{[1]}) \rangle_{^1S_0}^{\text{Born}}}{2N_c}, \quad (21a)$$

$$\Gamma_{\text{Born}}^{(v^2)}(^1S_0^{[1]} \rightarrow gg) = -\frac{2(2-\epsilon)}{3-2\epsilon} \frac{\mathbf{q}^2}{m_Q^2} \Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg), \quad (21b)$$

where $\Phi_{(2)} = \frac{1}{8\pi} (\frac{4\pi}{M^2})^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$ is the total two-body phase space in D dimension and $M = 2m_Q \sqrt{1 + \frac{\mathbf{q}^2}{m_Q^2}}$ is the quarkonium mass including the relativistic correction. The two Born diagrams are illustrated in Fig. 1.

The next-to-leading order calculations include real and virtual corrections. For S-wave Fock states (i.e. $^1S_0^{[1]}$ and $^1S_0^{[8]}$), UV divergences will be canceled by counter-term diagrams,

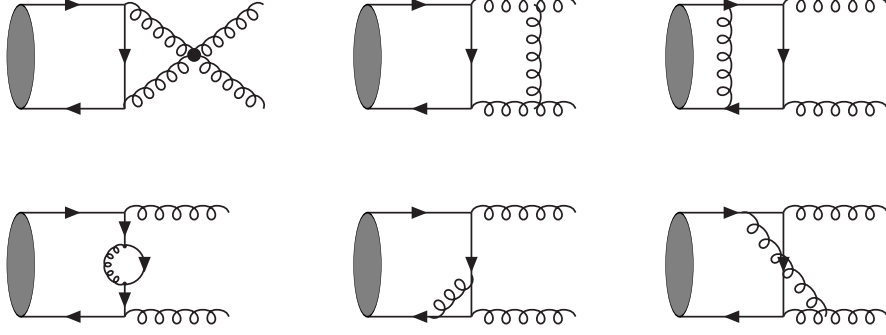


FIG. 2: Virtual correction Feynman diagrams for $^1S_0^{[1]} \rightarrow gg$. Only $^1S_0^{[8]}$ contributes for P -wave state.

and IR divergences will be canceled each other by the two corrections in the leading term in v^2 , but some soft divergences remain in the relativistic corrections. The cancelation of such additional divergences will be presented in next section by calculating NRQCD LDMEs at 1-loop level. The contribution of virtual plus counter-term corrections is

$$\begin{aligned} \Gamma_{\text{Virtual}}(^1S_0^{[1]} \rightarrow gg) = & \frac{3\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg) f_\epsilon(m_Q) \left\{ \left[-\frac{1}{\epsilon^2} - \frac{1}{6} \beta_0 \frac{1}{\epsilon} \right. \right. \\ & + \frac{1}{36} (-6\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 19\pi^2 - 44)] \\ & + \frac{\mathbf{q}^2}{m_Q^2} \left[\frac{4}{3} \frac{1}{\epsilon^2} - \frac{4n_f - 97}{27} \frac{1}{\epsilon} \right. \\ & \left. \left. - \frac{1}{324} (-72\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 8n_f + 267\pi^2 - 280) \right] \right\}, \end{aligned} \quad (22)$$

where $f_\epsilon(m_Q) = (\frac{\pi\mu_r^2}{m_Q^2})^\epsilon \Gamma(1+\epsilon)$. Some selected Feynman diagrams are shown in Fig. 2.

The real correction contains two sets, where the final states of one set is three gluons and the other one is $q\bar{q}g$. The diagrams are shown in Fig. 3 and Fig. 4 and the contributions to decay width are shown respectively as

$$\begin{aligned} \Gamma(^1S_0^{[1]} \rightarrow ggg) = & \frac{3\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg) f_\epsilon(m_Q) \left\{ \left[\frac{1}{\epsilon^2} + \frac{11}{6} \frac{1}{\epsilon} + \frac{1}{72} (724 - 69\pi^2) \right] \right. \\ & \left. + \frac{\mathbf{q}^2}{m_Q^2} \left[-\frac{4}{3} \frac{1}{\epsilon^2} - \frac{3}{\epsilon} - \frac{437 - 42\pi^2}{27} \right] \right\}, \end{aligned} \quad (23a)$$

$$\Gamma(^1S_0^{[1]} \rightarrow q\bar{q}g) = \frac{n_f}{2} \frac{\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg) \frac{f_\epsilon(m_Q)}{\Gamma(1+\epsilon)\Gamma(1-\epsilon)} \left[-\frac{2}{3} \frac{1}{\epsilon} - \frac{16}{9} + \frac{\mathbf{q}^2}{m_Q^2} \left(\frac{8}{9} \frac{1}{\epsilon} + \frac{86}{27} \right) \right]. \quad (23b)$$



FIG. 3: Real correction Feynman diagrams for $^1S_0^{[1]} \rightarrow ggg$. The crossed diagrams have been suppressed. It is the same for both $^1P_1^{[1]}$ and $^1S_0^{[8]}$ Fock states.

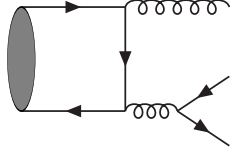


FIG. 4: Real correction Feynman diagrams for $^1S_0^{[1]} \rightarrow q\bar{q}g$. Only $^1S_0^{[8]}$ contributes for the P -wave state.

Combining Eqs. (21), (22) and (23), we obtain the hadronic decay width with both QCD radiative and relativistic corrections at NLO of 1S_0 heavy quarkonium,

$$\begin{aligned} \Gamma_{\text{QCD}}(^1S_0 \rightarrow \text{LH}) = & \Gamma_{\text{Born}}(^1S_0^{[1]} \rightarrow gg) \left\{ \left[1 + \frac{\alpha_s}{\pi} f_\epsilon(m_Q) \frac{1}{72} (-36\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) \right. \right. \\ & \left. \left. - 64n_f - 93\pi^2 + 1908) \right] - \frac{4}{3} \frac{\mathbf{q}^2}{m_Q^2} \left[1 + \frac{\alpha_s}{\pi} f_\epsilon(m_Q) \left(-\frac{4}{3} \frac{1}{\epsilon} \right. \right. \right. \\ & \left. \left. \left. + \frac{1}{144} (-72\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) - 164n_f - 237\pi^2 + 4964) \right) \right] \right\}. \end{aligned} \quad (24)$$

We note that our results agree with the previous work for $O(\alpha_s v^2)$ correction [12] and $O(\alpha_s)$ correction [8, 11]. By correctly comparing our results with Ref. [12], a slight difference of two body phase space Φ_2 between them can be found. In Ref. [12] Φ_2 is defined by removing the \mathbf{q}^2 dependence into the coefficients, so our individual virtual and real parts Eq. (22) and Eq. (23) look different from the results in Ref. [12] but essentially they are equivalent. The total NLO result Eq. (24) is explicitly the same, independent of the definition of Φ_2 . The correct repetition of the hadronic decay SD coefficients of 1S_0 heavy quarkonium enables our extended discussion from charm quark system to bottom quark system (i.e. η_b) and also partly checks our codes when dealing with P -wave heavy quarkonium.

B. Short-Distance Coefficients of P-Wave Quarkonium In Hadronic Decay

The procedure in calculating the hadronic decay width SD coefficients of 1P_1 heavy quarkonium is similar to but more complicated than the 1S_0 case. Additional simplification can be taken by imposing C (charge) parity conservation of QCD in selecting Feynman diagrams. A straightforward result is that C parity conservation prohibits $^1P_1^{[1]}$ Fock state, which has $C = -1$, to decay to two gluons, whose $C = +1$, no matter they are real or virtual. The results are shown as follows.

At the Born level,

$$\Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg) = \frac{5}{12} (4\pi\alpha_s)^2 \frac{\mu_r^{4\epsilon}}{m_Q^2} \Phi_{(2)}(1-\epsilon)(1-2\epsilon) \langle \mathcal{O}(^1S_0^{[8]}) \rangle_{^1P_1}^{\text{Born}}, \quad (25a)$$

$$\Gamma_{\text{Born}}^{(v^2)}(^1S_0^{[8]} \rightarrow gg) = -\frac{2(2-\epsilon)}{3-2\epsilon} \frac{\mathbf{q}^2}{m_Q^2} \Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg), \quad (25b)$$

For NLO corrections,

$$\begin{aligned} \Gamma_{\text{Virtual}}(^1S_0^{[8]} \rightarrow gg) = & \frac{3\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg) f_\epsilon(m_Q) \left\{ \left[-\frac{1}{\epsilon^2} + \frac{n_f - 21}{9} \frac{1}{\epsilon} \right. \right. \\ & + \frac{1}{72} (-12\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 29\pi^2 - 16)] \\ & + \frac{\mathbf{q}^2}{m_Q^2} \left[\frac{4}{3} \frac{1}{\epsilon^2} - \frac{4n_f - 115}{27} \frac{1}{\epsilon} \right. \\ & \left. \left. - \frac{1}{628} (-144\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 16n_f + 345\pi^2 - 992) \right] \right\}, \end{aligned} \quad (26)$$

$$\begin{aligned} \Gamma(^1S_0^{[8]} \rightarrow ggg) = & \frac{3\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg) f_\epsilon(m_Q) \left\{ \left[\frac{1}{\epsilon^2} + \frac{7}{3} \frac{1}{\epsilon} - \pi^2 + \frac{104}{9} \right] \right. \\ & \left. + \frac{\mathbf{q}^2}{m_Q^2} \left[-\frac{4}{3} \frac{1}{\epsilon^2} - \frac{4}{\epsilon} - \frac{554 - 45\pi^2}{27} \right] \right\}, \end{aligned} \quad (27)$$

$$\Gamma(^1S_0^{[8]} \rightarrow q\bar{q}g) = \frac{n_f}{2} \frac{\alpha_s}{\pi} \Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg) \frac{f_\epsilon(m_Q)}{\Gamma(1+\epsilon)\Gamma(1-\epsilon)} \left[-\frac{2}{3} \frac{1}{\epsilon} - \frac{16}{9} + \frac{\mathbf{q}^2}{m_Q^2} \left(\frac{8}{9} \frac{1}{\epsilon} + \frac{86}{27} \right) \right], \quad (28)$$

$$\begin{aligned} \Gamma(^1P_1^{[1]} \rightarrow ggg) = & \frac{40\alpha_s^3}{27} f_\epsilon(m_Q) (8\pi\Phi_2) \left\{ \left[-\frac{1}{\epsilon} + \frac{7\pi^2}{24} - \frac{5}{3} \right] \right. \\ & \left. + \frac{\mathbf{q}^2}{m_Q^2} \left[\frac{29}{15} \frac{1}{\epsilon} + \frac{4216 - 555\pi^2}{900} \right] \right\} \frac{\langle \mathcal{O}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}}}{2N_c m_Q^4}, \end{aligned} \quad (29)$$

Summing over the above results we get the total hadronic decay width,

$$\begin{aligned}
\Gamma_{\text{QCD}}(^1P_1 \rightarrow \text{LH}) = & \Gamma_{\text{Born}}(^1S_0^{[8]} \rightarrow gg) \left\{ \left[1 + \frac{\alpha_s}{\pi} f_\epsilon(m_Q) \left(-\frac{1}{2} \beta_0 \ln\left(\frac{4m_Q^2}{\mu_r^2}\right) \right. \right. \right. \\
& - \frac{8}{9} n_f - \frac{43\pi^2}{24} + 34 \left. \right] - \frac{4}{3} \frac{\mathbf{q}^2}{m_Q^2} \left[1 + \frac{\alpha_s}{\pi} f_\epsilon(m_Q) \left(-\frac{7}{12} \frac{1}{\epsilon} \right. \right. \\
& + \frac{1}{288} (-144\beta_0 \ln\left(\frac{4m_Q^2}{\mu_r^2}\right) - 328n_f - 735\pi^2 + 12304) \left. \right) \left. \right] \left. \right\} \quad (30) \\
& + \frac{40\alpha_s^3}{27} f_\epsilon(m_Q) (8\pi\Phi_2) \left\{ \left[-\frac{1}{\epsilon} + \frac{7\pi^2}{24} - \frac{5}{3} \right] \right. \\
& + \frac{\mathbf{q}^2}{m_Q^2} \left[\frac{29}{15} \frac{1}{\epsilon} + \frac{4216 - 555\pi^2}{900} \right] \left. \right\} \frac{\langle \mathcal{O}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}}}{2N_c m_Q^4}.
\end{aligned}$$

C. Evaluating NRQCD LDMEs And Matching Full QCD Results

In Eqs. (24) and (30) there exists explicit IR divergences. This is because, in principle we should evaluate both SD coefficients and LDMEs at the loop level and then take the matching procedure (10). So all the Born LDMEs appearing in Eq. (24) and (30) should be replaced by one-loop LDMEs by calculating the loop-corrections for them. Once we make such a replacement, all IR divergences should be canceled and the results become infra-red safe quantities. The self-energy contributions which connect Born LDMEs to their corresponding relativistic ones are first calculated in the Appendix of Ref. [1]. The intersecting diagrams which describe the E1 transition between $^1S_0^{[8]}$ and $^1P_1^{[1]}$ states at $O(\alpha_s v^2)$ in this work is new. The detailed calculation is presented in Appendix A. Here we give the relevant results in dimensional regularization and renormalize them in \overline{MS} scheme,

$$\langle \mathcal{O}(^1S_0^{[1]}) \rangle_{^1S_0}^{\text{Born}} \rightarrow \langle \mathcal{O}(^1S_0^{[1]}) \rangle_{^1S_0}^{(\mu_\Lambda)} \left\{ 1 - \frac{4}{3} \frac{\mathbf{q}^2}{m_Q^2} \frac{4\alpha_s}{3\pi} f_\epsilon(m_Q) \left[\frac{1}{\epsilon} - \ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right) \right] \right\}, \quad (31a)$$

$$\begin{aligned}
\langle \mathcal{O}(^1S_0^{[8]}) \rangle_{^1P_1}^{\text{Born}} \rightarrow & \langle \mathcal{O}(^1S_0^{[8]}) \rangle_{^1P_1}^{(\mu_\Lambda)} \left\{ 1 - \frac{4}{3} \frac{\mathbf{q}^2}{m_Q^2} \frac{7\alpha_s}{12\pi} f_\epsilon(m_Q) \left[\frac{1}{\epsilon} - \ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right) \right] \right\} \\
& + \frac{16\alpha_s}{9\pi} f_\epsilon(m_Q) \left\{ \left[\frac{1}{\epsilon} - \ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right) \right] \right. \\
& + \frac{3\mathbf{q}^2}{5m_Q^2} \left[-\frac{1}{\epsilon} + \ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right) \right] \left. \right\} \frac{\langle \mathcal{O}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}}}{2N_c m_Q^2}, \quad (31b)
\end{aligned}$$

$$\langle \mathcal{P}(^1S_0^{[8]}) \rangle_{^1P_1}^{\text{Born}} \rightarrow \langle \mathcal{P}(^1S_0^{[8]}) \rangle_{^1P_1}^{(\mu_\Lambda)} + \frac{16\alpha_s}{9\pi} f_\epsilon(m_Q) \left[\frac{1}{\epsilon} - \ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right) \right] \frac{\langle \mathcal{P}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}}}{2N_c m_Q^2} \quad (31c)$$

where μ_Λ is the factorization scale. Substituting them into Eq. (24) and (30), and considering the relation

$$\langle \mathcal{P}(^1S_0^{[1]}) \rangle_{^1S_0}^{\text{Born}} = \mathbf{q}^2 \langle \mathcal{O}(^1S_0^{[1]}) \rangle_{^1S_0}^{\text{Born}}, \quad (32a)$$

$$\langle \mathcal{P}(^1S_0^{[8]}) \rangle_{^1P_1}^{\text{Born}} = \mathbf{q}^2 \langle \mathcal{O}(^1S_0^{[8]}) \rangle_{^1P_1}^{\text{Born}}, \quad (32b)$$

$$\langle \mathcal{P}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}} = \mathbf{q}^2 \langle \mathcal{O}(^1P_1^{[1]}) \rangle_{^1P_1}^{\text{Born}}, \quad (32c)$$

we get the SD coefficients for heavy quarkonium hadronic decay of S-wave and P-wave states by matching full QCD and NRQCD,

$$F(^1S_0^{[1]}) = \frac{4\pi\alpha_s^2}{9} \left[1 - \frac{\alpha_s}{\pi} \frac{1}{72} (36\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 64n_f + 93\pi^2 - 1908) \right], \quad (33a)$$

$$G(^1S_0^{[1]}) = -\frac{4}{3} \frac{4\pi\alpha_s^2}{9} \left\{ 1 - \frac{\alpha_s}{\pi} \frac{1}{144} [192 \ln(\frac{\mu_\Lambda^2}{4m_Q^2}) + 72\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 164n_f + 237\pi^2 - 4964] \right\}, \quad (33b)$$

$$F(^1S_0^{[8]}) = \frac{5\pi\alpha_s^2}{6} \left[1 - \frac{\alpha_s}{\pi} \frac{1}{72} (36\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 64n_f + 129\pi^2 - 2448) \right], \quad (33c)$$

$$G(^1S_0^{[8]}) = -\frac{4}{3} \frac{5\pi\alpha_s^2}{6} \left\{ 1 - \frac{\alpha_s}{\pi} \frac{1}{288} [168 \ln(\frac{\mu_\Lambda^2}{4m_Q^2}) + 144\beta_0 \ln(\frac{4m_Q^2}{\mu_r^2}) + 328n_f + 735\pi^2 - 12304] \right\}, \quad (33d)$$

$$F(^1P_1^{[1]}) = \frac{5\alpha_s^3}{486} \left[7(\pi^2 - 16) - 24 \ln(\frac{\mu_\Lambda^2}{4m_Q^2}) \right], \quad (33e)$$

$$G(^1P_1^{[1]}) = \frac{\alpha_s^3}{3645} \left[1740 \ln(\frac{\mu_\Lambda^2}{4m_Q^2}) - 555\pi^2 + 9236 \right], \quad (33f)$$

where F 's and G 's are defined in Eq. (9a) and (9b).

The SD coefficients of $^1S_0^{[1]}$ agree with those in Refs. [1, 8, 11, 12, 16], that of $^1S_0^{[8]}$ and $^1P_1^{[1]}$ at leading order in v^2 are also agree with previous results in Ref. [8]. The relativistic corrections $G(^1S_0^{[8]})$ and $G(^1P_1^{[1]})$ are primarily new results in this work. Based on these results, we will further analyze the decay of 1S_0 and 1P_1 heavy quarkonium into light hadrons.

V. PHENOMENOLOGICAL DISCUSSIONS

A. Estimating NRQCD LDMEs

As shown in Eqs. (9a) and (9b) in previous section, the decay widths of heavy quarkonium are split into two parts, one is the SD coefficients which can be perturbatively calculated

in the framework of NRQCD factorization and the results are listed in Eq. (33), the other is LDMEs containing non-perturbative information and cannot be calculated precisely at present. For 1S_0 quarkonium there are two LDMEs and for 1P_1 are four. In Ref. [12] the LDMEs of η_c charmonium are determined by using the Cornell potential[20] and one of the experimental results of $\Gamma^{\text{LH}}(\eta_c)$ and $\Gamma^{\gamma\gamma}(\eta_c)$ [21], and then predict the other. In the present work, since there are no enough experimental information to determine all involved LDMEs, we will estimate the values of 1S_0 (i.e. η_b) and 1P_1 (i.e. h_c and h_b) LDMEs separately.

For η_b quarkonium, the situation is similar to Ref. [12], but lacking the experiment input of the decay width to two photons $\Gamma^{\gamma\gamma}(\eta_b)$. In this case we may first determine $\langle\mathcal{O}(^1S_0^{[1]})\rangle_{\eta_b}$ from potential model. Here we use the Buchmüller-Tye(B-T) potential model [22] and Cornell(Corn) potential model [20] results as input. They are [22, 23]

$$\langle\mathcal{O}(^1S_0^{[1]})\rangle_{\eta_b}^{\text{B-T}} = \frac{N_c}{2\pi}|R_S^{\text{B-T}}(0)|^2 = 3.093 \text{ GeV}^3, \quad (34a)$$

$$\langle\mathcal{O}(^1S_0^{[1]})\rangle_{\eta_b}^{\text{Corn}} = \langle\mathcal{O}(^1S_0^{[1]})\rangle_{\Upsilon(1S)}^{\text{Corn}} = 3.07_{-0.19}^{+0.21} \text{ GeV}^3. \quad (34b)$$

Here we use the heavy quarkonium spin symmetry, which holds in leading order of v , and higher order corrections for spin symmetry are expected to be small and less than the uncertainty of potential model estimates themselves, to relate η_b and $\Upsilon(1S)$. In order to determine $\langle\mathcal{P}(^1S_0^{[1]})\rangle_{\eta_b}$, we define [12, 16]

$$\langle\mathbf{v}^2\rangle_{\eta_b} \equiv \frac{\langle\mathcal{P}(^1S_0^{[1]})\rangle_{\eta_b}}{m_b^2\langle\mathcal{O}(^1S_0^{[1]})\rangle_{\eta_b}}, \quad (35)$$

although $\langle\mathbf{v}^2\rangle_{\eta_b}$ can not be understood as the expectation value of \mathbf{v}^2 in potential model, it can be estimated from the Gremm-Kapustin relation:

$$\langle\mathbf{v}^2\rangle_{\eta_b}^{\text{G-K}} = \frac{m_{\eta_b} - 2m_{\text{pole}}}{m_{\text{pole}}}. \quad (36)$$

Here we choose $m_{\text{pole}} = 4.6 \text{ GeV}$ for b quark and $m_{\eta_b} = 9401 \text{ MeV}$ from recent experimental measurement. Then we get $\langle\mathbf{v}^2\rangle_{\eta_b} = 0.044$, which is close to the potential model estimate $\mathbf{v}^2 \sim 0.05 - 0.1$.

For h_c quarkonium, the estimate is more complicated since we need to determine four LDMEs. $\langle\mathcal{O}(^1P_1^{[1]})\rangle_{h_c}$ is determined by the B-T potential model and

$$\langle\mathcal{P}(^1P_1^{[1]})\rangle_{h_c} \equiv \langle\mathbf{v}^2\rangle_{h_c} m_c^2 \langle\mathcal{O}(^1P_1^{[1]})\rangle_{h_c} \approx \langle\mathbf{v}^2\rangle_{\eta_c} m_c^2 \langle\mathcal{O}(^1P_1^{[1]})\rangle_{h_c}, \quad (37)$$

where $\langle \mathbf{v}^2 \rangle_{\eta_c} = 0.228$ from Ref. [12]. Here we have tentatively assumed $\langle \mathbf{v}^2 \rangle_{h_c} \approx \langle \mathbf{v}^2 \rangle_{\eta_c}$. The remaining two color-octet LDMEs are determined by the operator evolution method (OEM) [1, 24, 25]. From Eq. (31b) we get the evolution equations

$$\begin{aligned} \mu_\Lambda^2 \frac{d\langle \mathcal{O}(^1S_0^{[8]}) \rangle}{d\mu_\Lambda^2} &= -\frac{7\alpha_s}{9\pi} \frac{\langle \mathcal{P}(^1S_0^{[8]}) \rangle}{m_Q^2} + \frac{16\alpha_s}{9\pi} \frac{\langle \mathcal{O}(^1P_1^{[1]}) \rangle}{2N_c m_Q^2} - \frac{16\alpha_s}{15\pi} \frac{\langle \mathcal{P}(^1P_1^{[1]}) \rangle}{2N_c m_Q^4}, \\ \mu_\Lambda^2 \frac{d\langle \mathcal{P}(^1S_0^{[8]}) \rangle}{d\mu_\Lambda^2} &= \frac{16\alpha_s}{9\pi} \frac{\langle \mathcal{P}(^1P_1^{[1]}) \rangle}{2N_c m_Q^2}. \end{aligned} \quad (38)$$

by integrating them over the range from $\mu_{\Lambda_0} = m_c v \sim 0.8 \pm 0.2$ GeV to $2m_c$ for the typical charmonium NRQCD scales (here we neglect the initial values at $m_c v$ [25]). Here we set m_c to be its pole mass 1.5 ± 0.1 GeV, and the results are

$$\begin{aligned} \langle \overline{\mathcal{O}}(^1P_1^{[1]}) \rangle_{h_c} &\equiv \frac{\langle \mathcal{O}(^1P_1^{[1]}) \rangle_{h_c}}{2N_c m_c^4} = 3.537_{-0.805}^{+1.124} \text{ MeV}, \\ \langle \overline{\mathcal{P}}(^1P_1^{[1]}) \rangle_{h_c} &\equiv \frac{\langle \mathcal{P}(^1P_1^{[1]}) \rangle_{h_c}}{2N_c m_c^6} = 0.806_{-0.279}^{+0.428} \text{ MeV}, \\ \langle \overline{\mathcal{O}}(^1S_0^{[8]}) \rangle_{h_c} &\equiv \frac{\langle \mathcal{O}(^1S_0^{[8]}) \rangle_{h_c}}{m_c^2} = 2.514_{-0.724}^{+1.095} \text{ MeV}, \\ \langle \overline{\mathcal{P}}(^1S_0^{[8]}) \rangle_{h_c} &\equiv \frac{\langle \mathcal{P}(^1S_0^{[8]}) \rangle_{h_c}}{m_c^4} = 0.479_{-0.200}^{+0.311} \text{ MeV}. \end{aligned} \quad (39)$$

The above values meet the NRQCD velocity scaling rules and it is appropriate to determine CO matrix elements by OEM. The uncertainties are from variations of m_c , $\langle \mathbf{v}^2 \rangle$, $|R'_P(0)|_{h_c}$ for color-singlet(CS) matrix elements and also from the uncertainty of μ_{Λ_0} for color-Octet(CO) matrix elements. The related phenomenology about the uncertainties that might induce the variation of the decay width will be discussed in detail below.

Using the same method we can determine the LDMEs for h_b , they are

$$\begin{aligned} \langle \overline{\mathcal{O}}(^1P_1^{[1]}) \rangle_{h_b} &= 755.5_{-62.2}^{+69.4} \text{ keV}, & \langle \overline{\mathcal{P}}(^1P_1^{[1]}) \rangle_{h_b} &= 33.2_{-4.27}^{+4.90} \text{ keV}, \\ \langle \overline{\mathcal{O}}(^1S_0^{[8]}) \rangle_{h_b} &= 441.6_{-54.0}^{+63.6} \text{ keV}, & \langle \overline{\mathcal{P}}(^1S_0^{[8]}) \rangle_{h_b} &= 18.7_{-3.0}^{+3.5} \text{ keV}. \end{aligned} \quad (40)$$

Here we choose $m_b = 4.6 \pm 0.1$ GeV, NRQCD scale $\mu_{\Lambda_0} = m_b v \sim 1.5 \pm 0.2$ GeV and set $\langle \mathbf{v}^2 \rangle_{h_b} \approx \langle \mathbf{v}^2 \rangle_{\eta_b}$ for the same assumption as h_c . With these results of LDMEs, we can get the theoretical decay widths and compare them with the experimental data.

B. $\Gamma(\eta_b \rightarrow \mathbf{LH})$

With the LDMEs results given above (here the values $\langle \mathcal{O}(^1S_0^{[1]}) \rangle_{\eta_b}$ and $\langle \mathbf{v}^2 \rangle_{\eta_b}$), we can estimate the hadronic decay width of η_b . First we fix both the renormalization scale μ_r and

factorization scale μ_Λ to be $2m_b$ and consider the difference choice of LDMEs. At this scale the decay width can be rewritten as

$$\Gamma(\eta_b \rightarrow \text{LH}) = 441.1_{-5.9}^{+6.2} \times 10^{-3} \langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b} - 664.8_{-9.6}^{+9.2} \times 10^{-3} \langle \overline{\mathcal{P}}(1S_0^{[1]}) \rangle_{\eta_b}, \quad (41)$$

where

$$\begin{aligned} \langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b} &\equiv \frac{\langle \mathcal{O}(1S_0^{[1]}) \rangle_{\eta_b}}{2N_c m_b^2}, \\ \langle \overline{\mathcal{P}}(1S_0^{[1]}) \rangle_{\eta_b} &\equiv \frac{\langle \mathcal{P}(1S_0^{[1]}) \rangle_{\eta_b}}{2N_c m_b^4} = \langle \mathbf{v}^2 \rangle_{\eta_b} \langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b}. \end{aligned} \quad (42)$$

The LDME $\langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b}$ is determined by Eq. (34a), $\langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b}^{\text{B-T}} = 24.36_{-1.03}^{+1.09}$ MeV and $\langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b}^{\text{Corn}} = 24.18_{-1.81}^{+1.98}$ MeV. $\langle \overline{\mathcal{P}}(1S_0^{[1]}) \rangle_{\eta_b}$ is $\langle \mathbf{v}^2 \rangle_{\eta_b}$ smaller than $\langle \overline{\mathcal{O}}(1S_0^{[1]}) \rangle_{\eta_b}$ and get $\langle \overline{\mathcal{P}}(1S_0^{[1]}) \rangle_{\eta_b}^{\text{B-T}} = 1.07_{-1.06}^{+1.20}$ MeV and $\langle \overline{\mathcal{P}}(1S_0^{[1]}) \rangle_{\eta_b}^{\text{Corn}} = 1.06_{-1.05}^{+1.20}$ MeV.

These combinations of LDMEs give the hadronic decay width of η_b ,

$$\Gamma(\eta_b \rightarrow \text{LH})^{\text{B-T}} = 10.03_{-0.45}^{+0.48} \text{ MeV}, \quad (43a)$$

$$\Gamma(\eta_b \rightarrow \text{LH})^{\text{Corn}} = 9.96_{-0.83}^{+0.76} \text{ MeV}, \quad (43b)$$

and the μ_r dependence is plotted in Fig. 5. Here we see that the relativistic corrections only contribute about 10% to the total decay width and may not be important.

In order to compare with the experimental data of η_b total decay width [3], some approximation should be mentioned. The difference between the total decay width and the hadronic one of η_b in QCD is mainly from the process $\eta_b \rightarrow c\bar{c}X$, whose Feynman diagram are given by just replacing the final states $q\bar{q}$ pair in Fig. 4 by $c\bar{c}$ pair. Since it has no tree level contribution and is suppressed by the final state phase space, we can safely estimate the decay width of this process to be of the order $\alpha_s(1 - \frac{m_c}{m_b})^2 \sim 5\%$ less than the decay width into light hadrons. This value is within our error band and can omitted. So we have $\Gamma(\eta_b \rightarrow \text{LH}) \approx \Gamma^{\text{total}}(\eta_b) \sim 9 - 12.5$ MeV. This value is consistent with the experimental data $\Gamma^{\text{exp}}(\eta_b) = 10.8_{-3.7-2.0}^{+4.0+4.5}$ MeV [3].

C. $\Gamma(h_c \rightarrow \text{LH})$

The numerical values of SD coefficients for hadronic decay width of h_c are

$$\begin{aligned} \Gamma(h_c \rightarrow \text{LH}) &= 328.7_{-21.8}^{+26.1} \times 10^{-3} \langle \overline{\mathcal{O}}(1S_0^{[8]}) \rangle_{h_c} - 39.6_{-3.8}^{+3.1} \times 10^{-3} \langle \overline{\mathcal{O}}(1P_1^{[1]}) \rangle_{h_c} \\ &\quad - 446.0_{-35.5}^{+29.7} \times 10^{-3} \langle \overline{\mathcal{P}}(1S_0^{[8]}) \rangle_{h_c} + 92.4_{-7.3}^{+8.8} \times 10^{-3} \langle \overline{\mathcal{P}}(1P_1^{[1]}) \rangle_{h_c}, \end{aligned} \quad (44)$$

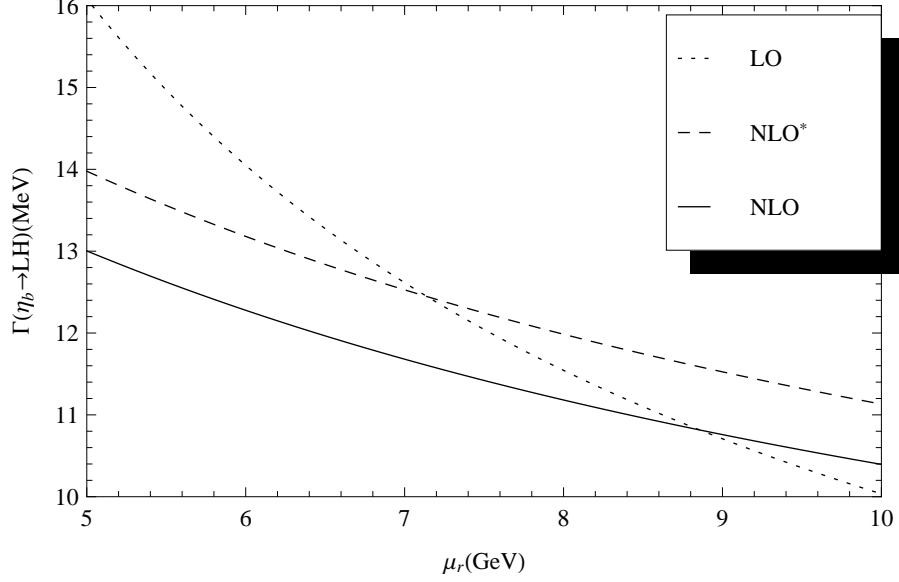


FIG. 5: μ_r dependence of $\Gamma(\eta_b \rightarrow \text{LH})$. LO represents values without QCD and relativistic corrections, NLO* includes QCD corrections but only at leading order in \mathbf{v} , and NLO takes into account all contributions at $O(\alpha_s v^2)$ including both QCD and relativistic corrections at next-to-leading order. The LDMEs are taken from the B-T potential model and the Gremm-Kapustin relation. Here μ_Λ is set to $2m_c$.

where we also set $\mu_r = \mu_\Lambda = 2m_c$ to eliminate logarithms and define the modified LDMEs as in Eq. (39). From these values we can see that the color-octet part, which contains larger SD coefficients, is the primary contribution to the decay width. We also note here that for $c\bar{c}$ system the relative velocity squared $\mathbf{v}^2 \sim 0.3$ is not small so the \mathbf{v}^2 power counting law does not work very well to classify the LDMEs. In fact we can see from Eq. (39) that the relativistic corrections are substantial as compared with the leading order contribution. The decay width including the total and each LDME contributions are listed in Table I.

TABLE I: $\Gamma(h_c \rightarrow \text{LH})$ expressed with the contributions of each LDME.

	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle_{h_c}$	$\langle \mathcal{O}(^1P_1^{[1]}) \rangle_{h_c}$	$\langle \mathcal{P}(^1S_0^{[8]}) \rangle_{h_c}$	$\langle \mathcal{P}(^1P_1^{[1]}) \rangle_{h_c}$	Total
$\Gamma(^{2S+1}L_J^{[c]} \rightarrow \text{LH})(\text{MeV})$	$0.83^{+0.37}_{-0.24}$	$-0.14^{+0.03}_{-0.05}$	$-0.21^{+0.09}_{-0.14}$	$0.07^{+0.04}_{-0.03}$	$0.55^{+0.27}_{-0.19}$

The results show that the relativistic correction is important for h_c hadronic decay, especially the color-octet correction. We can also see the contributions from different orders of α_s and \mathbf{v} , which are listed in Table II. The μ_r dependence are also shown in Fig. 6.

TABLE II: $\Gamma(h_c \rightarrow \text{LH})$ expressed with contributions at various orders of α_s and \mathbf{v} .

	$\alpha_s^0 v^0$	$\alpha_s^1 v^0$	$\alpha_s^0 v^2$	$\alpha_s^1 v^2$	Total
$\Gamma(h_c \rightarrow \text{LH})(\text{MeV})$	$0.40^{+0.18}_{-0.12}$	$0.29^{+0.17}_{-0.11}$	$-0.10^{+0.04}_{-0.07}$	$-0.04^{+0.03}_{-0.05}$	$0.55^{+0.27}_{-0.19}$

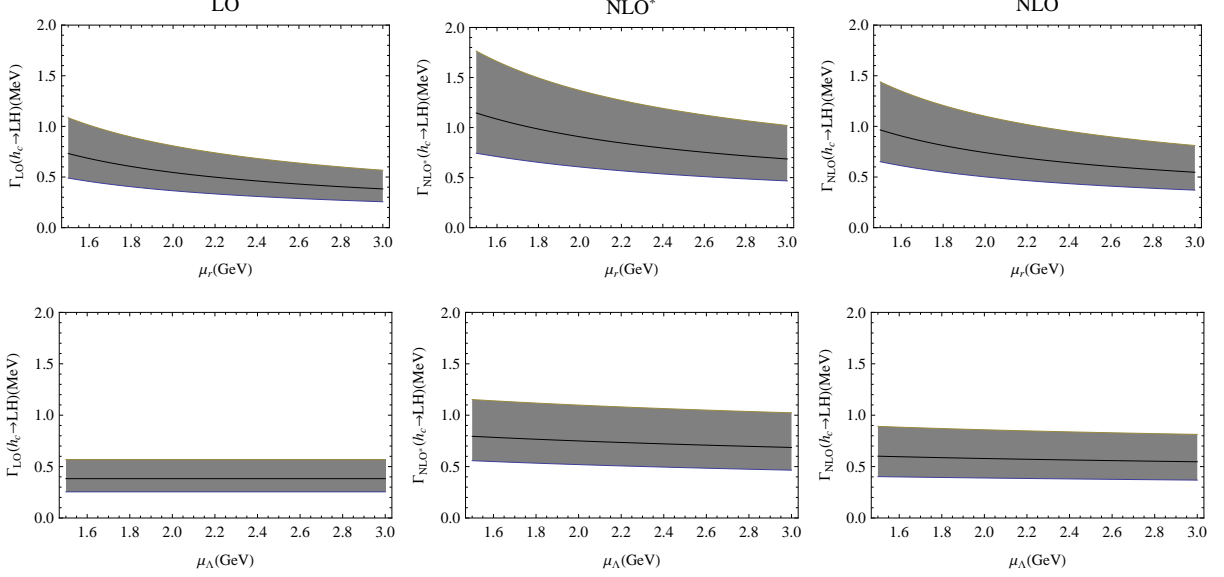


FIG. 6: μ_r and μ_Λ dependence of $\Gamma(h_c \rightarrow \text{LH})$. The upper plots are for μ_r and lower ones for μ_Λ . From left to right the plots are shown for LO, NLO* and NLO respectively, where NLO* includes $O(\alpha_s)$ but excludes $O(\alpha_s v^2)$ corrections.

Here we can see that the contributions of the first two terms to the decay width at leading order in \mathbf{v} are both positive, whereas the relativistic corrections are negative. These relativistic corrections play an important role and pull down the theoretical values of h_c decay width by partially canceling the contribution at leading order in \mathbf{v} . In order to compare with the experiment data [4], we should evaluate the E1 transition decay width $\Gamma(h_c \rightarrow \eta_c + \gamma)$ also up to \mathbf{v}^2 . Ref. [9] estimated the transition decay widths but only at leading order in v^2 by using the spin-symmetry relation between the spin-singlet and triplet P wave charmonia (weighted with a factor of $2J + 1$),

$$\Gamma(h_c \rightarrow \gamma \eta_c) = \frac{(E_\gamma^{h_c})^3}{9} \sum_{J=0}^2 (2J+1) \frac{\Gamma(\chi_{cJ} \rightarrow \gamma J/\psi)}{(E_\gamma^{\chi_{cJ}})^3}. \quad (45)$$

And the obtained E1 width is 615 ± 29 keV with the spin-triplet input in Eq. (45) extracted from PDG Data [21]. This result is consistent with the theoretical calculations at leading

order in v [42]. However, if we consider the v^2 corrections, the spin-symmetry relation will not hold at this order any more, and instead we should consider the relativistic corrections for wave functions. Ref. [42] showed that the width of $h_c \rightarrow \gamma\eta_c$ can be reduced from 650 KeV to 385 KeV by relativistic effects. Subsequent studies in various potential models [43–45] also observed these relativistic effects on this E1 transition width, ranging over 354–323 KeV. In this paper we choose the value $\Gamma(h_c \rightarrow \gamma\eta_c) = 385$ keV from Ref. [42].

These two decay channels (to LH and to $\gamma\eta_c$) are the dominant ones of h_c . The summation of the two contributions gives the theoretical total decay width $\Gamma^{\text{th}}(h_c) = 0.93_{-0.19}^{+0.27}$ MeV and the branching ratio $\mathcal{B}^{\text{th}}(h_c \rightarrow \eta_c + \gamma) = 41_{-9}^{+10}\%$. The total decay width falls in the error band of experimental data $\Gamma^{\text{exp}}(h_c) = 0.73_{-0.28}^{+0.45}$ MeV, while the branching ratio is a little lower than the experiment $\mathcal{B}^{\text{exp}}(h_c \rightarrow \eta_c + \gamma) = 54.3 \pm 6.7 \pm 5.2\%$. However, if we ignore the relativistic corrections to the hadronic decay width, the total width will increase to 1.07 MeV and the E1 transition branching ratio will be pulled down to 36%. Therefore, it is evident that the relativistic corrections play an important role in the h_c decay and can lead to a better agreement between theoretical estimate and the experimental data.

The uncertainties of hadronic decay width are mainly associated with the determination of color-octet(CO) matrix elements (see the second column in Table. 6). In OEM they are evaluated by integrating out the leading logarithms from μ_{Λ_0} to μ_Λ and are then proportional to $\ln(\frac{\alpha_s(\mu_{\Lambda_0})}{\alpha_s(\mu_\Lambda)})$. It is known that when the scale is close to Λ_{QCD} , the error of α_s becomes large. Therefore, the CO matrix elements are sensitive to the value of $\alpha_s(\mu_{\Lambda_0})$. In Fig. 7 we illustrate the μ_{Λ_0} dependence of hadronic decay width in comparison with μ_r, μ_Λ dependence in Fig. 6. We see that the decay width is indeed sensitive to the value of μ_{Λ_0} when $\mu_{\Lambda_0} < 1.5$ GeV. So we need to invoke other processes to determine it. If we assume that the initial scale μ_{Λ_0} is universal in all P-wave charmonia decays, we may determine it by e.g. $\chi_{cJ} \rightarrow \text{LH}(\gamma\gamma)(J = 0, 2)$, whose decay widths have been precisely measured, and this kind of studies may be left as our future work. The other underlying uncertainty is the color-singlet matrix element $\langle \overline{\mathcal{O}}(1P_1^{[1]}) \rangle_{h_c} \approx 3.5$ MeV which is determined by potential model in our calculations. We note that in Ref. [9] it was extracted from the data of χ_{cJ} decays and the value is 2.30 MeV, but this was the leading order result in v , where only two LDMEs are involved. At order v^2 many more LDMEs appear and need to be determined, which can not be extracted from limited experimental data at present. Nevertheless, if we take this smaller value 2.30 MeV as our input and keep relations Eqs. (37) and (38) unchanged, the hadronic decay

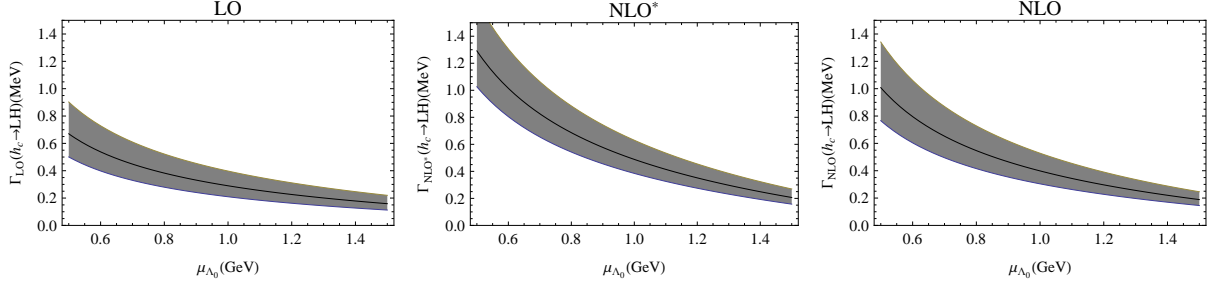


FIG. 7: μ_{Λ_0} dependence of $\Gamma(h_c \rightarrow \text{LH})$. From left to right the plots represent LO, NLO* and NLO respectively, where NLO* includes $O(\alpha_s)$ but excludes $O(\alpha_s v^2)$ corrections. Here μ_r and μ_Λ both are set to be $2m_c$.

width will be pulled down to $0.36^{+0.21}_{-0.15}$ MeV. Then the total decay width and E1 transition branching ratio will become $0.74^{+0.21}_{-0.15}$ MeV and $52^{+13}_{-11}\%$, which fit the experiment central value very well. Despite of all these uncertainties, we see that the negative contribution of relativistic corrections is certainly in favor of explaining the small decay width of h_c .

D. $\Gamma(h_b \rightarrow \text{LH})$

The analysis is by analogy with h_c decays except that the relativistic corrections are not so large for $b\bar{b}$ system as discussed in η_b decay. The SD coefficients and the decay width are

$$\begin{aligned} \Gamma(h_b \rightarrow \text{LH}) = & (150.2 \pm 2.1) \times 10^{-3} \langle \overline{\mathcal{O}}(1S_0^{[8]}) \rangle_{h_b} - (15.3 \pm 0.3) \times 10^{-3} \langle \overline{\mathcal{O}}(1P_1^{[1]}) \rangle_{h_b} \\ & - (203.3 \pm 2.9) \times 10^{-3} \langle \overline{\mathcal{P}}(1S_0^{[8]}) \rangle_{h_b} + (35.8 \pm 0.6) \times 10^{-3} \langle \overline{\mathcal{P}}(1P_1^{[1]}) \rangle_{h_b}, \end{aligned} \quad (46)$$

TABLE III: $\Gamma(h_b \rightarrow \text{LH})$ expressed with contributions of each LDME.

	$\langle \mathcal{O}(1S_0^{[8]}) \rangle_{h_b}$	$\langle \mathcal{O}(1P_1^{[1]}) \rangle_{h_b}$	$\langle \mathcal{P}(1S_0^{[8]}) \rangle_{h_b}$	$\langle \mathcal{P}(1P_1^{[1]}) \rangle_{h_b}$	Total
$\Gamma(^{2S+1}L_J^{[c]} \rightarrow \text{LH})(\text{keV})$	$66.33^{+9.60}_{-8.16}$	$-11.58^{+0.97}_{-1.08}$	$-3.81^{+0.61}_{-0.72}$	$1.19^{+0.18}_{-0.15}$	$52.13^{+8.62}_{-7.28}$

TABLE IV: $\Gamma(h_b \rightarrow \text{LH})$ expressed with various orders of α_s and v .

	$\alpha_s^0 v^0$	$\alpha_s^1 v^0$	$\alpha_s^0 v^2$	$\alpha_s^1 v^2$	Total
$\Gamma(h_b \rightarrow \text{LH})(\text{keV})$	$37.26^{+5.39}_{-4.57}$	$17.49^{+3.65}_{-3.04}$	$-2.11^{+0.34}_{-0.40}$	$-0.51^{+0.17}_{-0.21}$	$52.13^{+8.62}_{-7.28}$

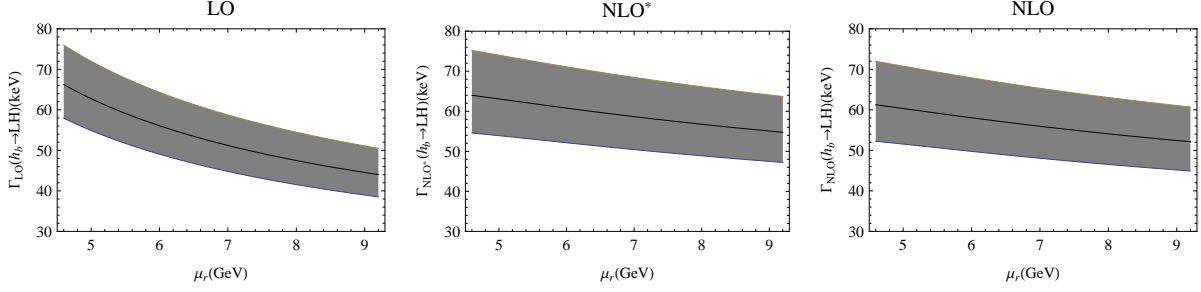


FIG. 8: μ_r dependence of $\Gamma(h_b \rightarrow LH)$. From left to right the three plots represent LO, NLO* and NLO respectively, where NLO* includes $O(\alpha_s)$ but excludes $O(\alpha_s v^2)$ corrections.

And the μ_r dependence is plotted in Fig. 8. From Table IV we see that the v^2 corrections are much smaller than the total decay width. The E1 transition decay width for h_b is evaluated in the NR [26], GI [44] and Screened-potential models [46]. The results are listed in Table V.

TABLE V: $\Gamma(h_b \rightarrow \eta_b + \gamma)$ and $\mathcal{B}(h_b \rightarrow \eta_b + \gamma)$ in NR, GI and Screened-potential models(SNR₀ is calculated using the zeroth-order wave functions while SNR₁ using the first-order relativistically corrected wave functions)

	NR	GI	SNR ₀	SNR ₁
$\Gamma(h_b \rightarrow \eta_b + \gamma)$ (keV)	41.8	37.0	55.8	36.3
$\Gamma_{\text{total}}(h_b)$ (keV)	93.9	89.1	107.9	88.4
$\mathcal{B}(h_b \rightarrow \eta_b + \gamma)$	44.5%	41.5%	51.7%	41.0%

Compared with the new experimental data $\mathcal{B}^{\text{exp}}(h_b(1P) \rightarrow \eta_b(1S)\gamma) = 49.2 \pm 5.7^{+5.6}_{-3.3}\%$ [3], the NR and SNR₀ results fit it well while the GI and SNR₁ ones are somewhat lower.

VI. SUMMARY

We have calculated order $\alpha_s v^2$ corrections for the annihilation hadronic decay widths of spin-singlet heavy quarkonia η_b , h_c and h_b within the framework of NRQCD. The short-distance (SD) coefficients are calculated by the method with matching procedure, and the long-distance matrix elements (LDMEs) are estimated by using the potential model and operator evolution methods. For the h_c decay, we find that $O(v^2)$ and $O(\alpha_s v^2)$ corrections

contribute large but negative values to the decay width, which substantially reduce the decay width calculated in the leading order in v^2 . It shows that relativistic corrections play an important role in hadronic decays of $c\bar{c}$ system, and can improve the theoretical results as compared with experimental data. Our calculated total decay width is $\Gamma^{\text{th}}(h_c) = 0.93^{+0.27}_{-0.19}$ MeV and the branching ratio $\mathcal{B}^{\text{th}}(h_c \rightarrow \eta_c + \gamma) = 41^{+10\%}_{-9\%}$ with the color-singlet matrix elements calculated by potential model, and $\Gamma^{\text{th}}(h_c) = 0.74^{+0.22}_{-0.15}$ MeV and $\mathcal{B}^{\text{th}}(h_c \rightarrow \eta_c + \gamma) = 52^{+13\%}_{-11\%}$ with the color-singlet matrix elements extracted from data (but at leading order in v) [9]. For η_b and h_b decays, we have calculated their hadronic decay widths and found that $\Gamma(\eta_b \rightarrow \text{LH}) \approx 9 \sim 12.5$ MeV depending on the choice of the LDMEs and $\Gamma(h_b \rightarrow \text{LH}) = 52.13^{+8.62}_{-7.28}$ keV. We conclude that for the $b\bar{b}$ system $O(\alpha_s v^2)$ corrections are not as important as in the $c\bar{c}$ system. We have also compared our theoretical results with experimental data [3, 4] and found that in general our calculations are consistent with data within theoretical and experimental uncertainties. In particular, we note that the operator evolution method used by us may provide a quite good estimate for the color-octet LDMEs.

Acknowledgments

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Appendix A: EVOLUTION OF NRQCD MATRIX ELEMENTS $\mathcal{O}(^1S_0^{[8]})$ and $\mathcal{P}(^1S_0^{[8]})$ AT $O(\alpha_s v^2)$

In order to cancel the infrared divergence in short-distance coefficients corresponding to $^1P_1^{[1]}$ Fock state, we should evaluate the NRQCD four-fermion operator $\mathcal{O}(^1S_0^{[8]})$ and $\mathcal{P}(^1S_0^{[8]})$ to sufficient orders.

The $O(\alpha_s)$ diagrams of LDMEs are sorted into three sets: self-energy diagrams which are related to self-energy corrections of external heavy (anti-)quarks, coulomb diagrams where the gluon is connected with both initial or final heavy quark and anti-quark and the

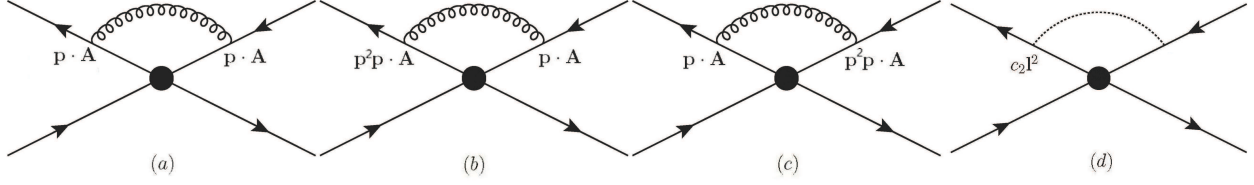


FIG. 9: The one-loop NRQCD diagrams which involve the Feynman rules up to $O(v^2)$. The Coulomb interactions and the cross diagrams have been suppressed.

intersecting diagrams where the gluon is related to an initial heavy (anti-)quark and a final (anti-)quark. The results of the first two sets have been given in Refs. [12, 27] and here we only calculate the intersecting diagrams which relate to the transition from S wave to P wave.

Using the Lagrangian shown in Eq. (3) and (5) we can write the amplitudes of each diagram, the integrals of the diagrams shown in Fig. 9 are (other crossed diagrams are not shown)

$$I_{a+b+c} = ig_s^2 \int \frac{d^D l}{(2\pi)^D} \frac{\mathbf{q} \cdot \mathbf{q}' - (\mathbf{q} \cdot \mathbf{l})(\mathbf{q}' \cdot \mathbf{l})/l^2}{m_Q^2(l_0^2 - \mathbf{l}^2 + i\epsilon)} \frac{1 - \mathbf{q}^2/2m_Q^2 - \mathbf{q}'^2/2m_Q^2}{[q_0 - l_0 - \frac{(\mathbf{q}-\mathbf{l})^2}{2m_Q} + i\epsilon][q'_0 - l_0 - \frac{(\mathbf{q}'-\mathbf{l})^2}{2m_Q} + i\epsilon]}, \quad (\text{A1a})$$

$$I_d = ig_s^2 \int \frac{d^D l}{(2\pi)^D} \frac{-1}{[q_0 - l_0 - \frac{(\mathbf{q}-\mathbf{l})^2}{2m_Q} + i\epsilon][q'_0 - l_0 - \frac{(\mathbf{q}'-\mathbf{l})^2}{2m_Q} + i\epsilon]}, \quad (\text{A1b})$$

where $q = (q_0, \mathbf{q})$ is the heavy quark external momentum and $l = (l_0, \mathbf{l})$ loop integral momentum. Since there is no pole on the upper-half of l_0 's complex plane, the second integral yields zero. Contour-integrating the first integral over l_0 around the $l_0 = |\mathbf{l}| - i\epsilon$ pole, we find

$$I_{a+b+c} = g_s^2 \int \frac{d^{D-1} l}{(2\pi)^{D-1}} \frac{\mathbf{q} \cdot \mathbf{q}' - (\mathbf{q} \cdot \mathbf{l})(\mathbf{q}' \cdot \mathbf{l})/l^2}{2m_Q^2 |\mathbf{l}|} \frac{1 - \mathbf{q}^2/2m_Q^2 - \mathbf{q}'^2/2m_Q^2}{[-|\mathbf{l}| - \frac{l^2}{2m_Q} + \frac{\mathbf{q} \cdot \mathbf{l}}{m_Q} + i\epsilon][-|\mathbf{l}| - \frac{l^2}{2m_Q} + \frac{\mathbf{q}' \cdot \mathbf{l}}{m_Q} + i\epsilon]}. \quad (\text{A2})$$

Here we should expand the relative momentum in the denominator. Assuming $\mathbf{q} \cdot \mathbf{l}/m_Q$,

$\mathbf{q}' \cdot \mathbf{l}/m_Q$ and \mathbf{l}^2/m_Q is far small than $|\mathbf{l}|$ [28], we get the required expansion

$$I_{a+b+c} = \frac{g_s^2}{2m_Q^2} \int \frac{d^{D-1}l}{(2\pi)^{D-1}} \frac{\mathbf{q} \cdot \mathbf{q}' - (\mathbf{q} \cdot \mathbf{l})(\mathbf{q}' \cdot \mathbf{l})/\mathbf{l}^2}{|\mathbf{l}|^3} (1 - \mathbf{q}^2/2m_Q^2 - \mathbf{q}'^2/2m_Q^2) \\ \times \left(1 + \left(\frac{\mathbf{q} \cdot \mathbf{l}}{|\mathbf{l}|m_Q} \right)^2 + \left(\frac{\mathbf{q}' \cdot \mathbf{l}}{|\mathbf{l}|m_Q} \right)^2 \right) + (\text{high order or irrelevant expansions}). \quad (\text{A3})$$

This integral can be reduced by taking the following substitution

$$\mathbf{l}^i \mathbf{l}^j \rightarrow \frac{1}{D-1} \delta^{ij} \mathbf{l}^2, \quad (\text{A4a})$$

$$\mathbf{l}^i \mathbf{l}^j \mathbf{l}^k \mathbf{l}^r \rightarrow \frac{1}{(D-1)(D+1)} (\delta^{ij} \delta^{kr} + \delta^{ik} \delta^{jr} + \delta^{ir} \delta^{kj}) \mathbf{l}^4, \quad (\text{A4b})$$

where δ^{ij} is $D-1$ dimensional Euclidean delta symbol. The integral yields

$$I_{a+b+c} = \frac{\pi \alpha_s^{(b)}}{2m_Q^2} \frac{\mathbf{q} \cdot \mathbf{q}'}{\pi^2} \frac{D-2}{D-1} \left(1 - \frac{D-1}{D+1} \frac{1}{2m_Q^2} (\mathbf{q}^2 + \mathbf{q}'^2) \right) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right). \quad (\text{A5})$$

Summing up all the diagrams we get

$$I = \frac{2\alpha_s^{(b)}}{\pi m_Q^2} \frac{D-2}{D-1} \mathbf{q} \cdot \mathbf{q}' \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \left(1 - \frac{D-1}{D+1} \frac{1}{2m_Q^2} (\mathbf{q}^2 + \mathbf{q}'^2) \right) \\ \times \left[C_F \frac{1 \otimes 1}{2N_c} + B_F T^a \otimes T^a \right] \mathcal{O}(^1S_0^{[8]}). \quad (\text{A6})$$

Recalling the definitions of $\mathcal{O}(^1P_1^{[1]})$ and $\mathcal{P}(^1P_1^{[1]})$ we can write

$$\langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle = \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle_{\text{Born}} + \frac{2(D-2)\alpha_s^{(b)}}{(D-1)\pi m_Q^2} \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \\ \times \left[C_F \frac{\langle H | \mathcal{O}(^1P_1^{[1]}) | H \rangle}{2N_c} - \frac{D-1}{(D+1)m_Q^2} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c} \right], \quad (\text{A7a})$$

$$\langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle = \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle_{\text{Born}} + \frac{2(D-2)\alpha_s^{(b)}}{(D-1)\pi m_Q^2} \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c}. \quad (\text{A7b})$$

Here we omit $\mathcal{O}(^1P_1^{[8]})$ and $\mathcal{P}(^1P_1^{[8]})$ coefficients since they are irrelevant in our case. The presence of UV divergence indicates that the LDMEs need renormalization. The relevant

counter-term in the \overline{MS} scheme can be chosen

$$\begin{aligned} \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle &= \mu_\Lambda^{-2\epsilon} \left\{ \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{UV}} + \ln 4\pi - \gamma_E \right) \right. \\ &\quad \times \left[C_F \frac{\langle H | \mathcal{O}(^1P_1^{[1]}) | H \rangle}{2N_c} - \frac{3}{5m_Q^2} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c} \right] \left. \right\}, \end{aligned} \quad (\text{A8a})$$

$$\begin{aligned} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle &= \mu_\Lambda^{-2\epsilon} \left\{ \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{UV}} + \ln 4\pi - \gamma_E \right) \right. \\ &\quad \times C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c} \left. \right\}, \end{aligned} \quad (\text{A8b})$$

where μ_Λ is the NRQCD renormalization scale. Combining Eq. (A7) and (A8) we find

$$\begin{aligned} \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle_{\text{Born}} &= \mu_\Lambda^{-2\epsilon} \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{IR}} + \ln 4\pi - \gamma_E \right) \\ &\quad \times \left(\frac{\mu}{\mu_\Lambda} \right)^{2\epsilon} \left[C_F \frac{\langle H | \mathcal{O}(^1P_1^{[1]}) | H \rangle}{2N_c} - \frac{3}{5m_Q^2} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c} \right], \end{aligned} \quad (\text{A9a})$$

$$\begin{aligned} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle_{\text{Born}} &= \mu_\Lambda^{-2\epsilon} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{IR}} + \ln 4\pi - \gamma_E \right) \\ &\quad \times \left(\frac{\mu}{\mu_\Lambda} \right)^{2\epsilon} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c}. \end{aligned} \quad (\text{A9b})$$

Combining the self-energy part (see Eq. (B14) in Ref. [1]) we get the total loop corrections of NRQCD LDMEs

$$\begin{aligned} \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle_{\text{Born}} &= \mu_\Lambda^{-2\epsilon} \langle H | \mathcal{O}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{IR}} + \ln 4\pi - \gamma_E \right) \\ &\quad \times \left(\frac{\mu}{\mu_\Lambda} \right)^{2\epsilon} \left[C_F \frac{\langle H | \mathcal{O}(^1P_1^{[1]}) | H \rangle}{2N_c} - \frac{3}{5m_Q^2} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c} \right. \\ &\quad \left. - \frac{N_c^2 - 2}{4N_c} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle \right], \end{aligned} \quad (\text{A10a})$$

$$\begin{aligned} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle_{\text{Born}} &= \mu_\Lambda^{-2\epsilon} \langle H | \mathcal{P}(^1S_0^{[8]}) | H \rangle^{(\mu_\Lambda)} + \frac{4\alpha_s}{3\pi m_Q^2} \left(\frac{1}{\epsilon_{IR}} + \ln 4\pi - \gamma_E \right) \\ &\quad \times \left(\frac{\mu}{\mu_\Lambda} \right)^{2\epsilon} C_F \frac{\langle H | \mathcal{P}(^1P_1^{[1]}) | H \rangle}{2N_c}. \end{aligned} \quad (\text{A10b})$$

These results are equivalent to Eq. (31) up to $O(\alpha_s v^2)$.

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